

Fraction Help

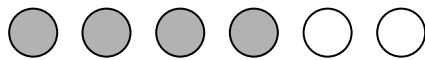
What is a Fraction?

A fraction is a number that names part of a whole. The whole can be a set or a region. The **numerator** of the fraction, which is the top number, names the “part”; whereas the **denominator** names the number of pieces in the whole.



This is an example of a region model. If you want to write a fraction comparing shaded pieces to whole pieces, the fraction would be $\frac{1}{4}$. In this example, “1” is the numerator, and “4” is the denominator.

This is an example of a set model. Each circle is a distinct part of the whole group.

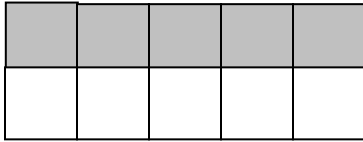


Here are several fractions that can be written using this set:

- shaded circles to the entire set of circles : $\frac{4}{6}$
- unshaded circles to the entire set of circles: $\frac{2}{6}$

Writing Equivalent Fractions

Equivalent Fractions are fractions that name the same amount.



In this example, $\frac{5}{10}$ pieces of the rectangle are shaded. Notice that $\frac{1}{2}$ of the entire rectangle is shaded so we can say $\frac{5}{10}$ is equivalent to $\frac{1}{2}$: $\frac{5}{10} = \frac{1}{2}$

A set of equivalent fractions can be created by multiplying or dividing both the numerator and denominator of a fraction by the same number.

Example: Suppose you wanted to name two fractions equivalent to $\frac{2}{5}$.

Pick any two numbers— In this example, we selected 3 and 6.

Equivalent Fraction #1: Multiply 2 and 5 by 3: $\frac{2}{5} \times 3 = \frac{6}{15}$

This means $\frac{2}{5} = \frac{6}{15}$

Equivalent Fraction #2: Multiply 2 and 5 by 6: $\frac{2}{5} \times 6 = \frac{12}{30}$

This means $\frac{2}{5} = \frac{6}{15} = \frac{12}{30}$

If a fraction has a **common factor**, in other words, a number that will divide evenly into both the numerator and denominator, you can perform that division to find another equivalent fraction.

Example: Megan completed 9 out of 12 problems, which can be expressed as

$\frac{9}{12}$. She notices that 9 and 12 have a common factor of 3 so that 9 and 12

can both be divided by 3: $\frac{9}{12} \div 3 = \frac{3}{4}$

This process of dividing a fraction by a common factor is also referred to as simplifying a fraction.



Simplifying Fractions/Lowest Terms

Simplifying fractions is the process of dividing a fraction by a common factor as in the previous example. However, a fraction is considered in **lowest terms**, which is often referred to as **simplest form**, when the numerator and denominator only have a common factor of 1. To express a fraction in lowest terms or simplest forms, simply divide both the numerator and denominator by the **greatest common factor**.

Example: Ken needs to find the simplest form of the fraction $\frac{16}{20}$. Ken realizes that 16 and 20 have these common factors:
1, 2, and 4.

To write $\frac{16}{20}$ in lowest terms (simplest form), Ken would divide both 16 and 20 by 4.

$$\begin{array}{l} \frac{16}{20} \div 4 = \frac{4}{5} \end{array}$$

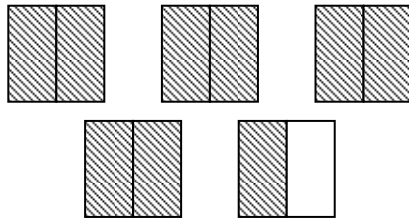
So, $\frac{16}{20}$ in simplest form is $\frac{4}{5}$. Notice, the only common factor between 4 and 5 is 1.

Mixed Numbers and Improper Fractions

An **improper fraction** is a fraction whose numerator is greater than or equal to its denominator. An improper fraction represents a quantity that is greater than or equal to one whole.

A **mixed number** is a number that combines a whole number with a fraction.

Example: Each square below is divided into half. If we count the number of shaded halves that are shown in this picture, we could count 9 and could express this quantity as $\frac{9}{2}$.



Notice that if we count again, we can count pieces that are shaded as we see 4

wholes and one $\frac{1}{2}$ piece shaded, which is $4\frac{1}{2}$.

$$\frac{9}{2} = 4\frac{1}{2}$$

When working with fractions, you will often be asked to express an improper fraction as a mixed number.

Example: Let's suppose you didn't have a picture of $\frac{9}{2}$ but needed to express this as a mixed number. Follow the following steps:

1. Divide the denominator into numerator. This whole number will be the whole number of the mixed number.
2. The remainder will be the numerator of the fraction.
3. The denominator will remain the same.



Sometimes, when working with fractions, you will need to express mixed numbers as improper fractions.

Example: If we didn't have the picture showing us that

$4\frac{1}{2} = \frac{9}{2}$ and we had to express $4\frac{1}{2}$ as an improper fraction, we could follow these steps:

1. Multiply the denominator by the whole number. [$2 \times 4 = 8$]
2. Add the numerator to this product. [$8 + 1 = 9$]
3. Place the sum obtained in step 2 over the original denominator of 2: $\frac{9}{2}$

Adding and Subtracting Fractions

When you add or subtract fractions, the denominators need to be the same. These denominators are referred to as "**like denominators**". For many problems, this already occurs.

Example: What is $\frac{4}{5} - \frac{1}{5}$?

To solve this problem, you would subtract the numerators [$4 - 1 = 3$] and place this difference of 3 over the "like denominator" of 5, which would be $\frac{3}{5}$.

Example: What is $\frac{1}{7} + \frac{2}{7}$?

Notice the "like denominators", which is 7. To solve this problem, add the numerators [$1 + 2 = 3$] and place this sum over 7. This problem has a sum of $\frac{3}{7}$.

Always check to make sure your answers are in simplest form!



How to find a Common Denominator: Adding and Subtracting with unlike Denominators

What happens when the denominators are not the same if you are adding or subtracting fractions?

If you have an addition or subtraction problem with unlike denominators, you will need to rename the fractions by finding a common denominator. Usually, we try to find the **least common multiple** of the denominators. The least common multiple is the smallest number both numbers will evenly divide into. This is usually referred to as **least common denominator** of the fractions.

Example: What is $\frac{2}{3} - \frac{1}{7}$?

Look at the two denominators: 3 and 7

Find a common denominator; 21 would be the least common denominator.

Write each fraction as an equivalent fraction using the denominator of 21:

$$\frac{2}{3} = \frac{?}{21} \quad \frac{1}{7} = \frac{?}{21}$$

To find the equivalent fraction for $\frac{2}{3}$ with a denominator of 21, you would multiply 2×7 , because $3 \times 7 = 21$. We always multiply or divide the numerator and denominator of fractions by the same number.

$$\begin{array}{l} \underline{2} \times 7 = \underline{14} \\ 3 \times 7 = 21 \end{array}$$

Now, write an equivalent fraction for $\frac{1}{7}$ with a denominator of 21. You would multiply 1×3 because $7 \times 3 = 21$.

$$\begin{array}{l} \underline{1} \times 3 = \underline{3} \\ 7 \times 3 = 21 \end{array}$$

Now, using the equivalent fractions of $\frac{14}{21}$ and $\frac{3}{21}$, complete the problem by subtracting the numerators: $14 - 3 = 11$, and place this difference over the denominator of 21: $\frac{11}{21}$



Adding and Subtracting Mixed Numbers

When you add and subtract mixed numbers, you need to follow these steps:

1. Add or subtract the fractions first. Make sure you have a common denominator. [Please note: Sometimes with subtracting fractions of mixed numbers you will need to regroup.]
2. Add or subtract the whole numbers next.
3. Make sure your fraction answer is in simplest form.

Here are some additional examples.

Example #1: What is the solution for this problem?

$$\begin{array}{r} 5\frac{8}{9} \\ + 2\frac{7}{9} \\ \hline \end{array}$$

1. We have a common denominator, so add $\frac{8}{9} + \frac{7}{9}$ is $\frac{15}{9}$.
2. Add the whole numbers, $5 + 2 = 7$.
3. The answer of $7\frac{15}{9}$ is not in simplest form, as the fraction is improper. Rewrite this improper as a mixed number of $1\frac{6}{9}$.
4. Combine the whole numbers of 7 and 1, which is 8. We know have $8\frac{6}{9}$, but $\frac{6}{9}$ is not in simplest form as 6 and 9 have a common factor of 3; divide 6 and 9 by 3 and the fraction is now $\frac{2}{3}$.
5. The final answer is $8\frac{2}{3}$.

Example #2: What is the solution for this problem? $7\frac{1}{2} - 4\frac{5}{6}$

1. Check to see if there is a common denominator, which does not happen in this problem. Find the LCD, which would be 6.

Rename the fraction $\frac{1}{2}$ as $\frac{3}{6}$.

2. Notice that the fraction part of the problem is: $\frac{3}{6} - \frac{5}{6}$. You cannot subtract 5 from 3, so you will need to "regroup" or "borrow".

3. To "regroup" take 1 whole from the 7, which means and add to the $\frac{3}{6}$. This means you are adding $1 + \frac{3}{6}$. Rename the 1 whole as $\frac{6}{6}$ so you have $\frac{9}{6}$. Don't forget the 7 is now a 6.

We now are working with $6\frac{9}{6} - 4\frac{5}{6}$.

4. Subtract the fractions: $\frac{9}{6} - \frac{5}{6} = \frac{4}{6}$

5. Subtract the whole numbers: $6 - 4 = 2$.

6. Make sure the answer of $2\frac{4}{6}$ is in simplest form. The fraction $\frac{4}{6}$ can be renamed as $\frac{2}{3}$ because 4 and 6 both divide evenly by 2.

7. Our final answer is $2\frac{2}{3}$.



How to Multiply Fractions

The neat thing about multiplying fractions is that you do not have to have common denominator. Here are the steps to follow when multiplying fractions:

1. Multiply the numerators.
2. Multiply the denominators.
3. Check to see if your answer is in simplest form.

Example: What is $\frac{1}{3} \times \frac{4}{5}$?

This is like working this problem: $\underline{1 \times 4}$

- 3 x 5
1. Multiply the numerators of 1 and 4: $1 \times 4 = 4$
 2. Multiply the denominators of 3 and 5: $3 \times 5 = 15$
 3. The product $\frac{4}{15}$ is in simplest form as the only common factor between 4 and 15 is 1.

How to Multiply Mixed Numbers

Here are the steps to follow when multiplying mixed numbers:

1. Rewrite the mixed numbers as improper fractions.
2. Multiply the numerators.
3. Multiply the denominators.
4. Check to see if your answer is in simplest form.

Example: What is $3\frac{2}{3} \times 2\frac{3}{4}$?

1. Rewrite $3\frac{2}{3}$ as $\frac{11}{3}$ and the $2\frac{3}{4}$ as $\frac{11}{4}$.

2. Our problem is $\frac{11}{3} \times \frac{11}{4} = \frac{121}{12}$.

3. Rename the improper fraction as a mixed number:

$$\frac{121}{12} = 10\frac{1}{12}$$



How to Divide Fractions

Dividing fractions involves an interesting procedure. You will need to rewrite a division of fraction problem as a multiplication problem. Here are the steps for dividing fractions:

1. Keep the first fraction the same.
2. Change the divide sign to a multiply sign.
3. Rewrite the second fraction in its **reciprocal form**.
[Turn the fraction “upside down”. For example the reciprocal of $\frac{4}{5}$ is $\frac{5}{4}$. A reciprocal of a number is a number that you can multiply with that number to get the answer of 1 whole.]
4. Follow the steps for multiplying fractions.

Example: What is $\frac{7}{8} \div \frac{2}{3}$?

1. Rewrite the problem as $\frac{7}{8} \times \frac{3}{2}$
2. Multiply: $\frac{7 \times 3}{8 \times 2}$ which = $\frac{21}{16}$
3. Rename $\frac{21}{16}$ as $1\frac{5}{16}$.



How to Divide Mixed Numbers

When you divide mixed numbers, you will follow these steps:

1. Rename each mixed number as an improper fraction.
2. Follow the steps listed for dividing fractions.

Example: What is $6\frac{3}{5} \div 2\frac{1}{2}$?

1. Rename $6\frac{3}{5}$ as $\frac{33}{5}$ and $2\frac{1}{2}$ as $\frac{5}{2}$.

2. This problem can now be thought of as $\frac{33}{5} \div \frac{5}{2}$.

3. Rewrite this as a multiplication problem: $\frac{33}{5} \times \frac{2}{5}$

4. This results in $\frac{66}{25} = 2\frac{16}{25}$.